MODELING CHANGE USING STELLA

Contents

Introducción

Stella is a dynamic modeling system in which relational models are built by creating a pictorial diagram of a system and then assigning the appropriate values and functions to it.

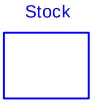
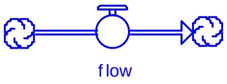
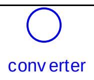
After starting the program, find the 🌍 button in the left vertical toolbar and click it to change it into a x^2 symbol. This starts the interactive model-building layer. Otherwise, you won't be able to input formulas or quantities into the diagram.

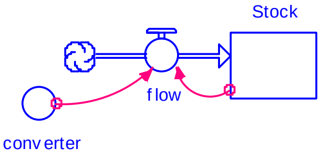
There are several tools available for use in creating a Stella model. Click on them once in the menu bar and place them in the model by clicking again where you want them to appear.

Stella es un sistema de modelamiento dinámico en el cual pueden construirse modelos relacionales creando diagramas gráficos del sistema, y asignando los valores y funciones apropiadas en el. Las principales herramientas para crear un modelo en Stella son cuatro, y se explican en la Tabla 1. Existen muchos otros componentes y funciones en Stella, sin embargo con los anteriores es suficiente para ilustrar nuestros cálculos (para mayores referencias véase por ejemplo High Performance Systems, 2001).

Tabla 1
Componentes básicos de un sistema dinámico en Stella

Table 1
Basic component of a dynamic system in Stella

Explicación	Símbolo
Los montos son llamados stocks, y representan acumulaciones. Estos montos están influenciados por flujos de entrada y/o flujos de salida. Un ejemplo es el saldo en la cuenta bancaria.	
Los Flujos están definidos por una tasa (monto por unidad de tiempo). Los flujos influyen en los Stocks causando acumulaciones y/o agotamientos. Ejemplo: el interés periódico que gana una cuenta bancaria.	
Los Converters son usados para ingresar parámetros o constantes al sistema, o para hacer operaciones aritméticas, conversión de unidades, u otras necesidades matemáticas.	

Los Connectors llevan flujos de información entre los componentes, y están representados por una flecha. En el ejemplo de la derecha, un flujo se acumula continuamente en un stock, y el stock usa como información un parámetro de entrada proporcionado por un converter, y la información del mismo stock.	

Graph This button opens up a graph. Double-click on the graph to bring up a window that allows you to select which inputs to display on the graph.

The other buttons on the top toolbar are explained in “Help.” Leaving your mouse pointer over the each icon momentarily displays a short comment about what the button’s function.

1. Rate of Change – What Stella Models

Many of the situations we want to model involve modeling the way a quantity changes over time. Often what we know or can observe is the *rate* at which the quantity changes. Let's define some notation that will be useful.

$Q(t)$: The quantity that we want to examine (e.g., U.S. population, position of a base ball, amount of product in a chemical reaction) at a particular time, t. $Q(0)$ is the initial amount we have available, $Q(2)$ would be the amount after 2 time units, etc.

$\frac{\Delta Q}{\Delta t} = \frac{\text{change in } Q}{\text{change in time}}$: The observed, measured or estimated change in behavior of Q over a set time interval. We will assume that these measurements are taken at uniform time steps so that all of the time intervals are the same, say 1 time unit or .25 time units, etc.

Many models of change involve building for the rate of change a mathematical expression – algebraic, graphical, tabular – that mimics the behavior of the actual system being studied. Stella allows us to input this sort of information and then run a simulation to see how the quantity we are interested changes.

2. Example Model – Single Species Population Dynamics

We will begin by modeling the change in the population of a single species over time. Let's suppose we have a field full of rabbits. We'll let

$P(t)$ = the population of rabbits at time t.
There are 10 rabbits to start, so $P(0) = 10$.

Before Modeling: world mode and construction view

To build a model in Stella, we need to be sure we are at the right level and in the right mode. Start Stella; you should see a blank page. At the left side there should be an up and down arrow and a world icon. We are in the *world view* of the *model building level*.

We can start building the model in this view, but eventually we will need to get into the *construction view* by pressing on the globe – it will change to a X^2.

When in **construction mode**, we can input relationships and values (i.e., build the functioning model); when in **world mode**, we can annotate each part of the model, adding an explanation so that others can see why we set things up the way we did.

At this middle, modeling building, level we have all of the tools needed to build and test our model. Across the top are the model elements (stocks, flows, converters, connectors) along with the output tools (graphs, tables, numeric displays).

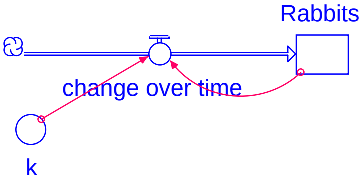
Let's build some models.

Model 0: “the more the merrier”

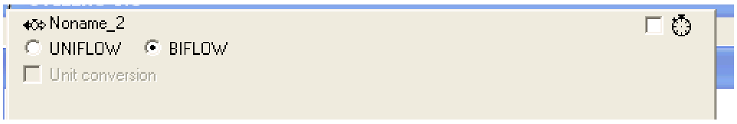
In this most simple model, we assume that when more rabbits present, more rabbits will be born without considering death. More mathematically, the change in P over time is proportional to the number of rabbits present:

$$\frac{\Delta P}{\Delta t} = kP;$$
$$P(0) = 10.$$

We’ll look at the solution to this problem over time using the second case – the *difference* equation. We can draw the dependencies in this equation as follows:



Es importante que el flujo sea modificado de 'Uniflow' a 'biflow' (the flow now has a shaded arrow at the end opposite the rabbit stock), para que el flujo sea lo suficiente general como para permitir que ingresen flujos negativos al stock (i.e., para permitir que los conejos mueran):



Noname_2
○ UNIFLOW ● BIFLOW
☐ Unit conversion

Notice that we have a one directional “inflow” in this model, so we are assuming that rabbits don’t die for now. Before we begin to investigate this model, we need to talk about how to estimate the parameter, k. The first question is, *what exactly does k tell us?* Look at it again this way:

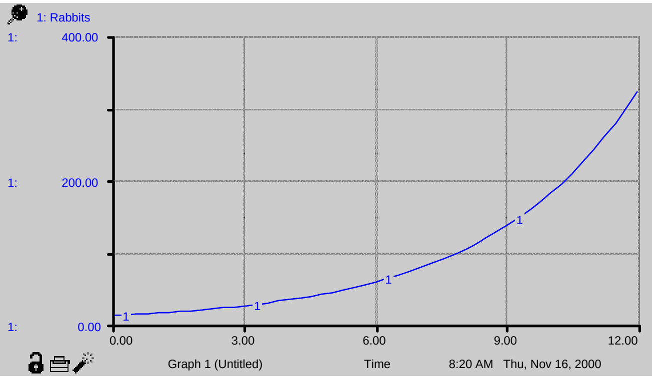
$$\frac{\text{change in } P}{P} = k$$

The change in population per unit of population is equal to k. In other words, k tells us how fast the population is changing at any given population level. So a good description of k is that it is the *rate of growth* of the population (or of decrease if k is negative). If we look at k as a fraction, for example 0.5 or one half, we can say that at each time step there is a new rabbit for every two rabbits already in the population at the beginning of that time step. Obviously, k is dependent on the time step we are using; for example if the time step is in minutes, k is probably smaller than one half.

In general, parameter estimation is a difficult problem when dealing with biological or sociological models (rather than models from physics, where some measurement can often give us the parameter value). We will use the value 0.3 for now for *k*, keeping in mind that an actual value for *k* would involve field work and observations, collecting data to use to estimate an appropriate growth factor.

We'll model this in Stella as follows:

1. Build the dependency diagram in the construction (middle) level, just as it looks in the diagram above – A stock for the rabbits, a converter for the constant, *k*, and a flow for the change of the rabbits over time.
2. Enter the rate (*k*) in the converter named k by double clicking on the converter; let's use 0.3. (Be sure that you are in editing mode rather than the world view mode.)
3. Enter the initial data (population = 10) in the rabbit stock.
4. Build a formula for change in the population over time: k * rabbits.
5. Pin down a graph and set it to plot rabbits over time.



This model makes sense, at least early in the time span for our population. However, as time continues on, this model has an exponential growth. (Why? Solve this simple differential equation – only calculus is required.[1]) As time gets larger the population grows without bound, clearly a limitation of this model. How can we fix this unrealistic exponential growth? Let's add death.

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[1] The equation $\frac{dP}{dt} = kP$ can be separated to form: $\frac{dP}{P} = kdt$. Integrating both sides gives: $\ln P = kt + C$. Solving this for P by exponentiating both sides gives:

$$P = e^{kt+C} = \left(e^{C}\right)e^{kt} = Ae^{kt}.$$

Notice that using the usual convention, the *e* to a constant power (hence a constant) was renamed as *A*.

Model 0.5: Adding Death proportional to the number of rabbits.

The simplest way to add death is to think of it in the same way as we did birth in model 0: The more rabbits the more crowded, so the more likely a rabbit will die.

$$\frac{\Delta P}{\Delta t} = kP - dP\,;\;\; P(0) = 10\,.$$

This is a good idea, but notice that we will have the same model as before:

$$\frac{\Delta P}{\Delta t} = kP - dP = (k - d)P = \hat{k}P\,.$$

Again we would have the change equal to a constant times *P*. Which would yield the same result at Model 0. What else can we try?

Model 1: Competition between individuals

Let's let the death term be proportional to the number of possible 2-rabbit interactions. Using a combinatorial argument, we can figure out this number:

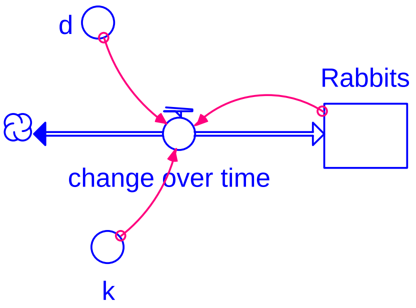
There are *P* possible rabbits to choose for interacting rabbit number one and *(P-1)* possible rabbits for interacting rabbit number two – and which order the rabbits are chosen shouldn't matter – giving us $P\ (P-1)\ /\ 2$ for the number of two rabbit interactions.[2]

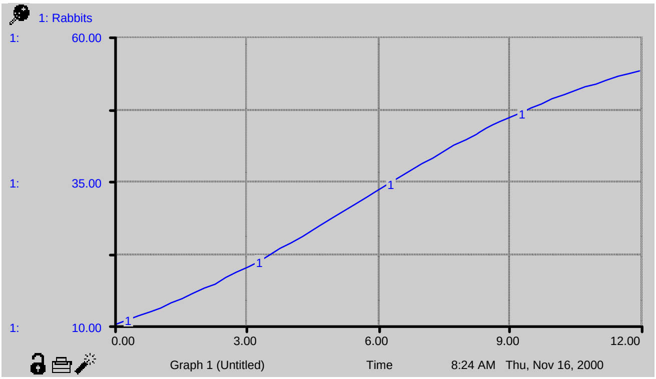
Putting this all together we have:

$$\frac{\Delta P}{\Delta t} = kP - d\frac{P(P-1)}{2};\ P(0) = 10\,.$$

Let's modify our stella model:

- Modify the dependency diagram to include a converter for the new parameter, *d*.
- Enter the rate (*d*) in the converter named d – let's use 0.01.
- Change the formula for change in the population over time:
  k * rabbits – d*rabbits*(rabbits-1)/2.
- Check the formulas in the lowest level.





This is more realistic behavior – the population levels off over time – in this case at around 60. What other ways can we incorporate death?

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[2] This counting principle can also be obtained from the more general situation of choosing 2 objects from *P* objects and using the combinatorial formula:

$$\binom{P}{2} = \frac{P!}{2!(P-2)!} = \frac{P(P-1)}{2}$$

Model 2: Maximum Sustainable Population

Let's assume that, rather than fighting over scarce resources as in the two-rabbit interaction model above, the species we are studying has less offspring when resources are scarce, and hence the model should reflect a slowing birth rate as some maximum population is approached.

Consideremos que se tiene una ecuación de comportamiento de una población de conejos (P) a través del tiempo, en la que la tasa de crecimiento de la población (k) depende de cuan cerca esté dicha población del número máximo capaz de soportar el medio ambiente (máxima capacidad sostenible) de conejos (M), en el sentido de que la tasa de crecimiento cae en la medida de que la población se aproxime a M. La población inicial de conejos es P(0)=10. La ecuación diferencial es escrita:

$$\frac{\Delta P}{\Delta t} = k\left(1 - P/M\right)P$$
$$P(0) = 10$$

El modelo en Stella, con una población máxima de M=100 conejos y k=30%, puede ser representado por el Diagrama 1. En este caso, inicialmente el stock 'conejos' contiene el número inicial de 10, mientras que M contiene la capacidad máxima del medio de 100 conejos, y K contiene un componente de la tasa de crecimiento igual a 30%. El flujo contiene la ecuación dada por K*(1-Conejos/M)*Conejos.
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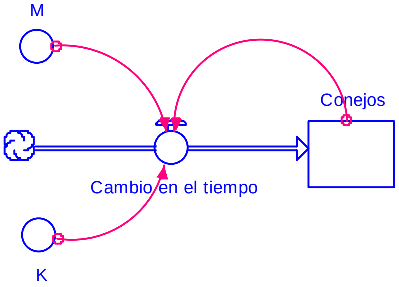


Diagrama 1: Modelo básico de una población de conejos en Stella

Al realizar las iteraciones en Stella, estableciendo previamente un horizonte de tiempo máximo de T=200, el resultado de la simulación, es decir la solución de la ecuación diferencial, se muestra en el Gráfico 1, donde se aprecia la forma en que la población de conejos converge al punto de máxima sostenibilidad a través del tiempo.

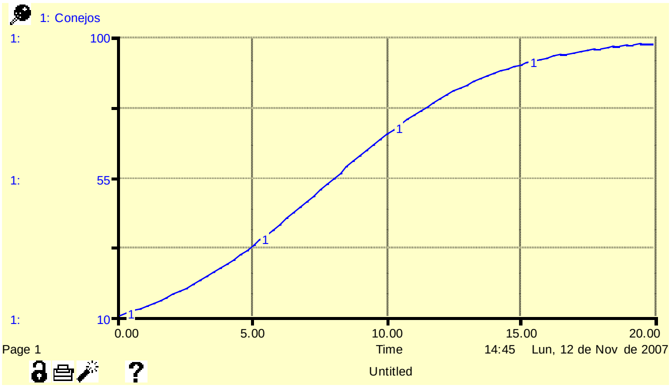


Gráfico 1: Comportamiento de la población de conejos a través del tiempo

This model gives the same rough shape as the two-rabbit interaction model. This is not a coincidence. Look at the algebraic simplifications of the right hand sides of the two equations – both are in the form constant times *P* minus constant times *P* squared; in effect, the same model.

3. Systems Models -- Interacting Species

Now we want to look at two species that interact: *x* is the prey and *y* is the predator. Now we have two equations instead of one, each with increase rate and decrease rate terms.

$$\frac{\Delta x}{\Delta t} = in_x - out_x \qquad\qquad \frac{\Delta y}{\Delta t} = in_y - out_y$$

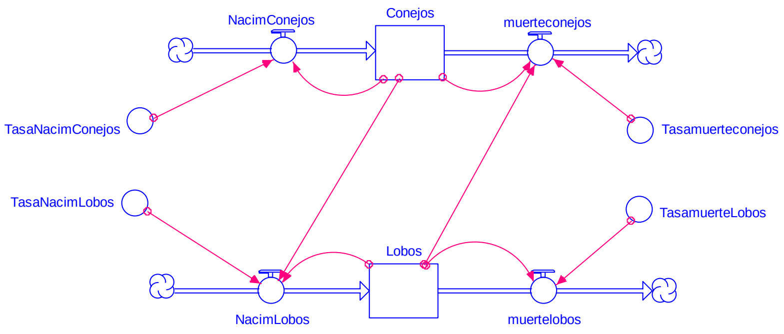
So how should we model these four terms? Here are some ideas -- in historical order.

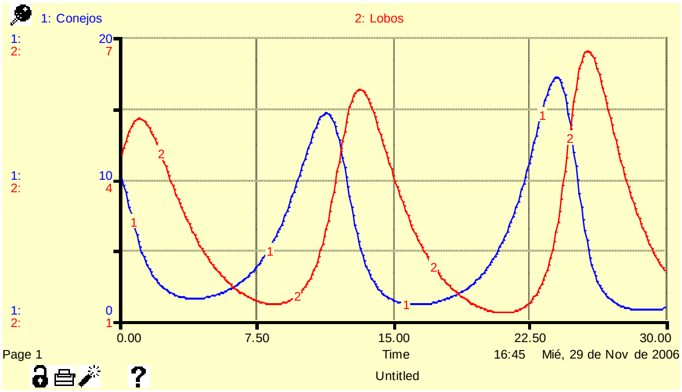
3.1 Original Lotka-Volterra Model (Volterra 1927)

$$\frac{\Delta x}{\Delta t} = b_x x - d_x xy \qquad\qquad \frac{\Delta y}{\Delta t} = b_y xy - d_y y$$

Volterra set the prey (*x*) increase to simple Malthusean and had the predator (*y*) affect the prey through the death term. He set the birth term of the predator proportional to both the predators present and the prey present, since predators would have a hard time reproducing without food. Notice the four parameters: b_x , b_y, d_x, and d_y. These can be interpreted as the population birth rates and death rates for each of the species.

Suppose the predator species is wolves and the prey species is rabbits, and let us start with 10 rabbits, 4 wolves, rabbit birth and death rates of 0.7 and 0.3, and wolf birth and death rates of 0.08 and 0.44. Using Stella, con td=0.1, we get:





Notice that the populations cycle, with the predator peaking right after the prey.

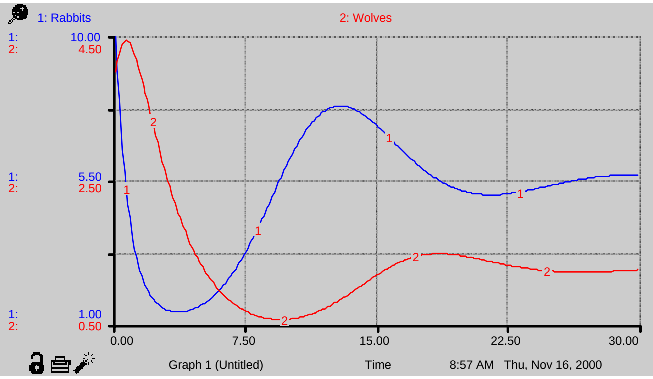
3.2. Competition in the Prey (1930s)

The primary objection to the LV Model was that the prey (*x*) would increase without bound if the predators died out.

What if competition among the prey is also incorporated, using a maximum sustainable prey population (*M*)?

$$\frac{\Delta x}{\Delta t} = b_x\left(1 - {}^{x}\!/_{M}\right)x - d_x xy \qquad \frac{\Delta y}{\Delta t} = b_y xy - d_y y$$

Here is the Stella graph with the maximum population set to M=12.



Notice that the populations cycle, with the predator peaking right after the prey, but the cycles die off, approaching a stable population.

3.3. Leslie (1950s)

Leslie wanted to add more realism into the model. He had two main objections:

- There is no upper limit to the relative rate of increase of predator
- Predator should do worse as the predator to prey ratio increases

Leslie fixed these by changing the death term for the predator to have both the number of predators and the ratio of predators to prey.

$$\frac{\Delta x}{\Delta t} = b_x x - d_x xy \qquad \frac{\Delta y}{\Delta t} = b_y xy - d_y y \frac{y}{x}$$

Again, the populations cycle, with the predator peaking right after the prey, but the cycles die off even more quickly, approaching a stable population.

3.4. May (1960s)

May liked both the Leslie model and the competition model and combined them, but also noted the following:

The prey death term implies that for a given *y*, the number of prey eaten is proportional to the number of prey present.

This implies that predators are never not hungry. He fixed this by adding a piece to the prey death that would control this term.

$$\frac{\Delta x}{\Delta t} = b_x \left(1 - {}^{x}\!/\!_{M}\right) x - d_x y \frac{x}{1+x}$$

$$\frac{\Delta y}{\Delta t} = b_y xy - d_y y \frac{y}{x}$$

4. Interacting Systems Problems

Leslie and May Population Models:

Finish the development of Leslie's and May's models by implementing them in Stella, using the same parameter values as in the other models. Explain why in May's model the prey death term now allows for wolves to become satiated by looking at the limit of this term as *x* gets very large, but *y* stays fixed. Comment on the similarities and differences in behavior between May and the other models.

Rabbits-Wolves-Grass:

Add grass to your predator-prey model. Think about how the rabbits would eat the grass and how the grass would grow back.

Predator-Prey Parameter Experimentation:

For each of the models above, experiment with changing each of the parameters and record your observations. Write if summary of your findings from these experiments, and comment on whether your observations make sense given what you expected. Do all of the parameters seem to govern the same qualitative behavior regardless of the particular model?

Phase Plane Graphs:

Another graph that can be examined in the case where there are two independent variables x and y is the *phase plane* which is the scatterplot of *x* versus *y*. Generate these graphs, and look at what information is conveyed by them.

Two Species Competition:

In the situation where we have two species who compete for the same resources (rather than one being the food source for the other), we can build a two equation model in a similar fashion. The birth terms do not involve interaction between the species (i.e., only *x* is present in the birth term for the *x* equation), but the death terms for each involve both species. Build a model that reflects this situation. What are the parameters? What units are involved in each term? Model this in Stella. What happens to the populations? Experiment with parameter values and report your findings. Beware: This model will exhibit very strange behavior!

Cooperation:

In the situation where we have two species whose survival depends on their mutual cooperation, we can build a two equation model in a similar fashion. The death terms would not involve interaction (i.e., only *x* is present in the death term for the *x* equation), but the birth terms for each involve both species. Build a model that reflects this situation. What are the parameters? What units are involved in each term? Model this in Stella. What happens to the populations? Experiment with parameter values and report your findings.

Military Battles:

In a battle, we can model the actions (and subsequent army strengths) in the same way as we did for predator-prey interaction, except that the "birth terms" -- which are now better called "reinforcement terms" -- would not involve any interaction, and the "death terms" -- which might still be called death terms -- are really only dependent on the strength of the opposing army. Build a model that reflects this situation. What are the parameters? What units are involved in each term?
(a) Model this in Stella. What happens to the armies? Experiment with parameter values and report your findings.
(b) Army X is about to attack army Y. Army Y has 1000 troops and army X has 3000 troops, but army Y has superior weaponry and training, making each Y soldier 1.65 more effective than an X soldier. This can be interpreted as the time it takes an X to kill a Y being 1.65 times longer. Model this situation and comment on who "wins."

Modeling the Economy:

Here is an idea for a simple model for an economy in which all means of production are socially owned. Consider the following assumptions (originally proposed by G. A. Feldman):

- The economy is divided into two sectors, Producer goods (where goods which will be used or invested in both sectors are produced), and Consumer goods (where goods which will be consumed by the population are produced).
- The annual rate of output from each sector is proportional to the amount invested in that sector. The constants of proportionality can be assumed to be the same for each sector – the reciprocal of this constant is called the *marginal capital coefficient*.
- The output from the producer sector is split in some proportion and invested in both the producer and the consumer sectors. The output from the consumer sector is consumed (i.e., not invested).

Develop a model and provide an analysis of and explanation for all parameters used. What are relative rates of growth of the sectors and the national income tending toward over time? (relative rate of growth can be thought of as rate of change relative to current size)

Epidemics:

Another common situation which can be modeled using a system of equations is a general model for a rapidly spreading epidemic, in which persons who get the disease die from it (e.g., the ebola virus). We need a model that incorporates the following assumptions:

- Healthy people get sick at a rate proportional to both the number of healthy people and the number of sick people.
- Sick people die at a constant rate.

(a) Develop such a model and provide an analysis of and explanation for all parameters.
(b) What if the disease allows for people to recover and then be immune (for example, measles)? Then we would need to consider three populations: Susceptible, Infected, and Recovered (SIR).

Model this situation thinking in the following way: The number of susceptible people is declining at a rate proportional to the number of infected people, the number of infected people is growing at a rate proportional to the number of times infected people come in contact with susceptible people with people who are recovering being subtracted out of the infected pool, and the number of recovered people is growing at a rate proportional to the number of infected people. Develop such a model and provide an analysis of and explanation for all parameters used.

Two Tank Blending:

Two large tanks, each holding 10 liters of a saltwater solution are interconnected by pipes. Fresh water flows into tank 1 at a rate of 5 liters per minute, and fluid is drained out of tank B at the same rate. The tanks have two pipes connecting them which allow for exchange of fluid at the following rates: from A to B at 7 liters per minute and from B to A at 3 liters per minute. If the solution in tank A contains 40 grams of salt, and the solution in tank B contains 80 grams of salt, model the changes in the concentrations in the tanks over time. Experiment with your model by varying the parameters and recording your results.

Two Zone Heating:

A house consists of two heating zones: The main living area (zone A) and the bedroom wing (zone B). The living area is heated by a furnace, but the bedroom wing gets all of its heat through transfer through the walls. Suppose the time constant (1/k) for the heat transfer between zone A and zone B is 2 hr and the time constant for heat transfer to the outside is 5 hr. If the outside temperature is 0°, how cold can it get in the bedroom wing?

Great Lakes Pollution:

Model the flow of pollution in the Great Lakes given the following information:

Lake	Volume	Water in	Water out
Superior	2900 mi^3	15	15 (to Huron)
Michigan	1180 mi^3	38	38 (to Huron)
Huron	850 mi^3	15+38+15	68 (to Erie)
Erie	116 mi^3	68+17	85 (to Ontario)
Ontario	393 mi^3	85+14	99

Flow rates are given in mi^3/year. Assume that there is no "back flow" between the lakes.

(a) Assume the lakes are currently clean, and experiment with placing a source of pollution on the banks of Superior. If the pollution sources dump at a constant rate, how long will it take for all of the lakes to be heavily polluted?

(b) Assume the lakes are currently polluted, and determine how long it would take for the pollution level to be reduced by 50% if only clean water is flowing into the lakes from the streams and rivers. How long would it take for the pollution to be reduced to 5% of its original level?

Competing Companies:

A simple model of competing companies can be obtained from the following assumptions:

- Let *x(t)* and *y(t)* represent the quarterly profits for each company at time *t*.
- The change in the quarterly profit for each can be broken into the rate at which money flows in and the rate at which money flows out.
- For each company, the rate at which money flows in is proportional to its current profits, and the rate at which money flows out is proportional to its competitor's profit.

Build a model using these assumptions, and experiment with the parameters. Look at relative sizes of the parameters, and comment on the behavior of the model.

5. Stella como Integrador Numérico

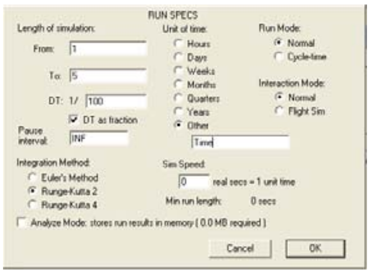
Sabemos que en tiempo continuo los costos de producción son una corriente o caudal continuo de dinero Y(t), una función el tiempo. Si el intervalo de tiempo bajo estudio va de desde t=0 hasta t=T, el valor presente del flujo viene dado por la integral definida:

$$VP = \int_{t=0}^{t=T} Y(t)e^{-it}dt$$

Example 1 Evaluate $\int_1^5 3x^2\,dx$. Since the indefinite integral is $x^3 + c$, this definite integral has the value

$$\int_1^5 3x^2\,dx = x^3\Big]_1^5 = (5)^3 - (1)^3 = 125 - 1 = 124$$

Usando un dt suficientemente pequeño (por ejemplo 0,01), un rango entre 1 y 5, y seleccionando el método de Runge-Kutta:



RUN SPECS

Length of simulation:
From: 1
To: 5
DT: 1/ 100
☑ DT as fraction
Pause interval: INF

Unit of time:
○ Hours
○ Days
○ Weeks
○ Months
○ Quarters
○ Years
● Other
Time

Run Mode:
● Normal
○ Cycle-time

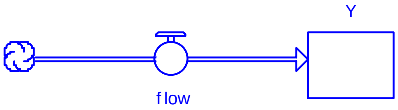
Interaction Mode:
● Normal
○ Flight Sim

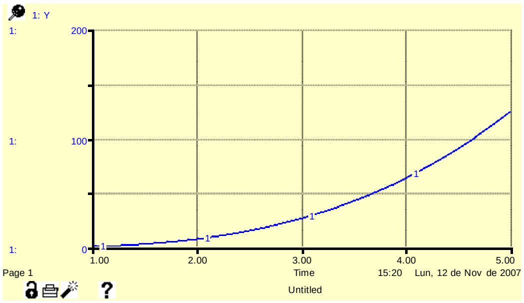
Integration Method:
○ Euler's Method
● Runge-Kutta 2
○ Runge-Kutta 4

Sim Speed
0 real secs = 1 unit time
Min run length: 0 secs

☐ Analyze Mode: stores run results in memory (0.0 MB required)

ponemos en el flujo: 3*Time*Time, y en el stock Y=0, entonces se obtiene:

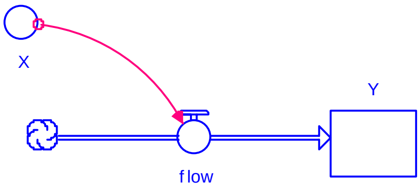




Ejemplo 2: Evaluar $\int_a^b ke^x dx$. Note que esto corresponde aproximadamente a un valor futuro acumulado del caudal o flujo. Los resultados de la integración son:

$$\int_a^b ke^x dx = ke^x\Big]_a^b = k(e^b - e^a).$$

Supongamos que k=$100, a=0 años y b=10 años, entonces 100*(e^{10}-e^{0})= $ 2.202.546,58. En Stella el diagrama es igual al anterior, con la diferencia que ponemos en el flujo: 100*exp(Time), y en el stock Y=0, para los límites 0 a 10. También puede escribirse como:



Poniendo en X=100, y en el flujo X*exp(time).

Sin embargo, en muchos casos prácticos las funciones de comportamiento pueden llegar a ser complejas y muy difíciles de trabajar algebraicamente como en el caso anterior, y en estos casos Stella cumple un papel valioso como simulador e integrador numérico. Para ilustrar eso a continuación se proponen y resuelven dos ejemplos prácticos: un proyecto de inversión acuícola, y el caso de una decisión de inversión en educación superior.

Ilustración 1

Asumamos que se está evaluando un proyecto acuícola, para lo cual se debe estimar el VAN a un horizonte de 10 años (t=120 meses). El proyecto consiste en adquirir la tecnología necesaria para engordar larvas de camarón, para lo cual ser requiere una inversión inicial de $1 000 000. La tasa de descuento continua es del 2% mensual. El proyecto presupuesta ingresos sólo al momento de la venta (cosecha) dados por el peso (en gramos) $W_t=500*t*ln(50+t^2)$ a un precio de $20 el gramo. Los costos operacionales (de alimentación y almacenamiento) se estiman continuos y constantes a través del tiempo en $10 000 mensuales.

El Diagrama 2 muestra la organización del modelo en Stella. En la nomenclatura de Stella cada componente con tiene lo siguiente:

```
peso' = 500*time*LOGN(50+time*time);
VP Ingresos = Peso*20*exp(-tasa*time);
tasa = 0.02;
costo = 10000*exp(-tasa*time);
VP Costo =0, en Inversion = 1000000,
VAN = -Inversion - VP Costo + VP Ingresos.
```

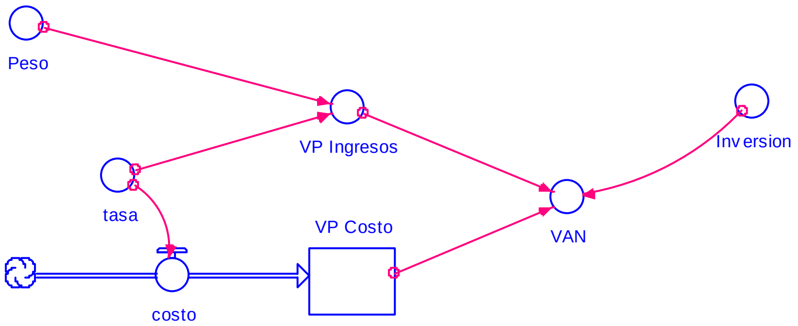


Diagrama 2: Modelo acuícola básico en Stella

Luego de efectuar la simulación en Stella se aprecia en el Gráfico 2 que el VAN a los 120 meses es negativo por algo más de $400 000[3], por lo que debe rechazarse la inversión. Sin embargo el gráfico muestra también que hay un periodo de tiempo en la que se espera un VAN positivo, y esto ocurre aproximadamente si el proyecto finalizara entre los meses 40 y 80. En efecto, analizando las tablas de salida de Stella (no mostradas aquí) es posible concluir con mucha claridad el momento

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[3] En efecto: $$VAN = -1\,000\,000 + 20 \times 500 \times t \times ln(50 + 120^2)\exp(-0.02 \times 120) - \int_{t=0}^{t=120} 10\,000\exp(-0.02t)\,dt = -411916.35$$

exacto en términos que la decisión correcta no es cosechar el mes 120 sino que el mes 56, cuando el VAN es de $137027.

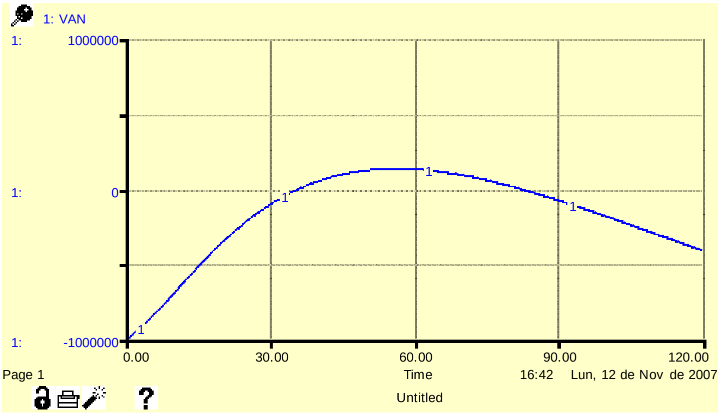


Gráfico 2: VAN del proyecto acuícola como función del mes de término del proyecto.

Ilustración 2

A continuación mostramos el ejemplo de la inversión en educación desarrollado por Henderson y Quandt (1985). La situación es que una persona al terminar la enseñanza secundaria, en t=0, debe decidir entre entrar comenzar a trabajar inmediatamente, o continuar su educación para comenzar a trabajar una vez terminados los estudios. Los flujos de ingreso en ambos casos duran hasta su retiro en T=50 años, que es el horizonte de evaluación. Si se entra inmediatamente a trabajar su corriente de ingresos es $g(t)=2400e^{0.08t}$, pero y si va a la universidad, es $f(t)=800e^{0.12t}$.

La primera pregunta a responder es cuál es la mejor decisión si la tasa de descuento es 10%. Una vez creado el modelo en Stella y realizadas las simulaciones, el Gráfico 3 muestra el VAN de cada alternativa. Se aprecia que en este ejemplo el VAN de realizar estudios universitarios es siempre menor que en el caso de comenzar a trabajar inmediatamente terminada la enseñanza secundaria.

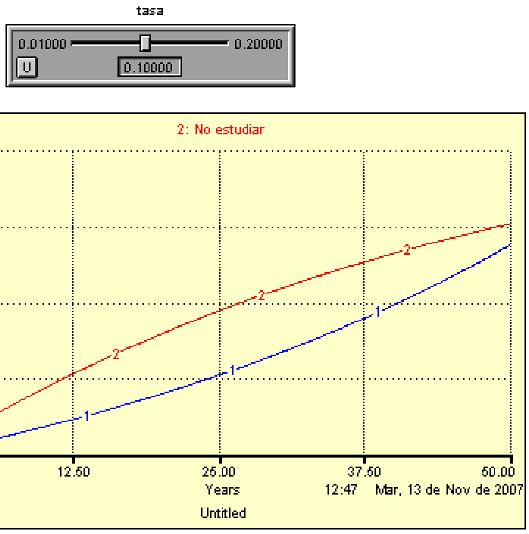


Gráfico 3: Comparación de VAN: comenzar a trabajar vs. seguir estudiando (i=10%)

Sin embargo, si bien parece no ser conveniente ir a la universidad a la tasa de descuento del 10%, surge la pregunta ¿a qué tasa de descuento se estaría indiferente entre ambas alternativas?, o alternativamente, para qué tasas de descuento convendría seguir estudiando. Para esto se estima el VAN diferencial entre ambas opciones y se iguala a cero, para a continuación buscar la tasa de descuento que resuelve dicha ecuación, es decir:

$$VAN = \int_{t=0}^{t=50} (800e^{0.12t} - 2400e^{0.08t})e^{-it}dt =$$

$$800\left[\frac{e^{(6-50i)} - 1}{0.12 - i} - \frac{3\left(e^{(4-50i)} - 1\right)}{0.08 - i}\right] = 0$$

Resolviendo iterativamente, con una tasa i=0.088 (8.8%) se cumple la condición que el VAN=0, y por lo tanto, se concluye que la educación universitaria es una inversión favorable si las tasas de descuento son menores que 8.8%. El siguiente gráfico muestra que efectivamente a dicha tasa de interés el VAN de ambas opciones coincide en un horizonte de 50 años. También el gráfico muestra que a esta tasa de descuento la diferencia (negativa) entre ambas opciones es máxima alrededor del año 30.

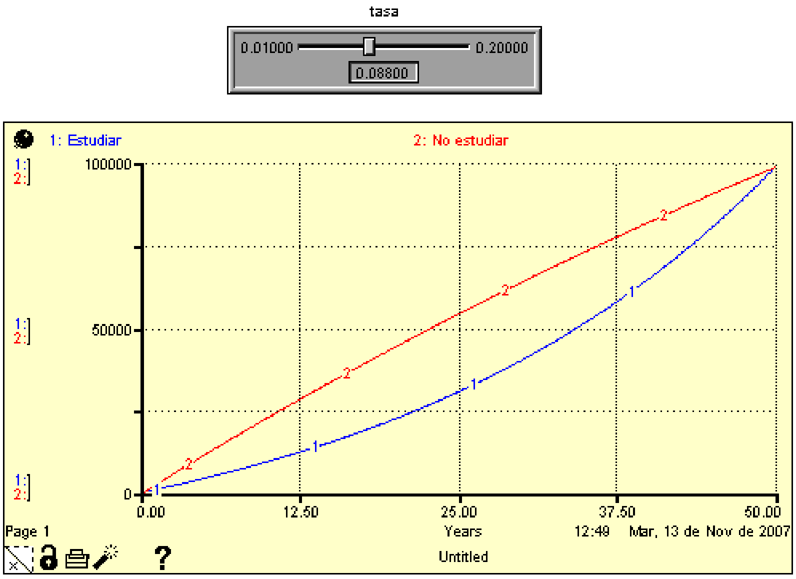


Gráfico 4: Comparación de VAN: comenzar a trabajar vs. seguir estudiando (i=8.8%)

Finalmente, el Gráfico 5 muestran tres simulaciones comparativas sensibilizando el VAN diferencial de ambas alternativas (es decir VAN de estudiar menos VAN de no estudiar) para tres tasas de descuento: (1) al 7.6%, (2) al 8.8% y (3) al 10%. Los resultados permiten apreciar de otro modo que efectivamente dicho VAN diferencial es positivo para los casos (1) y (2), y que si el horizonte de evaluación se extendiese algunos años más hasta T=55, las tres opciones arrojan un resultado positivo.

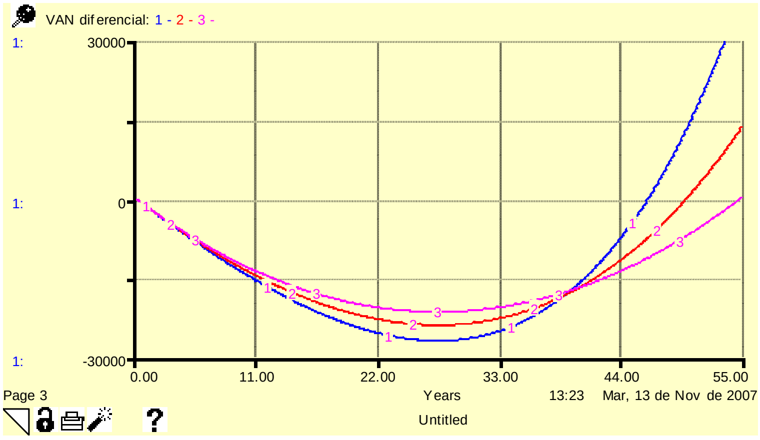
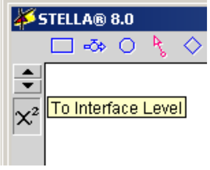


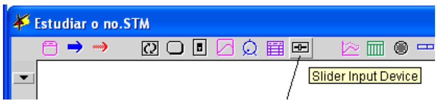
Gráfico 5: VAN diferencial: comenzar a trabajar vs. seguir estudiando: (1) i=7.6%, (2) i=8.8%, (3) i=10%.

6. What if: slider bar feature

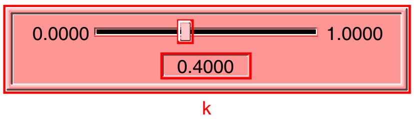
Once we have a method for producing a graphical or tabular solution, we can ask “what if” questions to test the realism of the model and the sensitivity of the model to changes in the parameters.

Stella provides an excellent environment for this testing, especially through the **slider bar feature**. At the interface (top) level we can add a slider bar and attach it to the parameter *k* in our simple model. Go to the top level by clicking the up-arrow at the upper left of the window; there will be a blank interface page.





Notice that the tools across the top have changed slightly, and now among other things, a slider bar is available. Pull one down.



To connect the slider bar to the converter containing *k*, double click on the slider bar and choose k from the list. Notice that the range of allowable values can be set in this dialog window as well. Unfortunately, slider bars are not available at the middle work level, so running the model will now take place at the upper level. Graphs and tables for the output are available at this level as well. The problems below involve modifying the model and investigating realism and sensitivity.

7. Stella Avanzado….(working…..)

Given this pattern for the outflow from *Mature Trees*, the map "tells you" that the pattern over time traced by the stock will be completely determined by what happens to the *becoming mature* flow. Do you "hear" this?

If the *becoming mature* flow steps-up at the same time as the *harvesting* flow, the *Mature Trees* stock will remain unchanged; i.e., inflow and outflow will remain equal. Hence, the magnitude of the stock will not change.

But does the *becoming mature* flow step up at the same time as the *harvesting* flow?

No! For six years after the step-increase in *harvesting* occurs, the *becoming mature* flow will remain equal to the pre-step *harvesting* rate. That's because there is six year's worth of seedlings that are "in development," and the number of seedlings in each year's cohort is equal to the value of the pre-step *harvesting* rate. So, six years *after* the step increase in harvesting occurs, the *becoming mature* flow will finally step-up to equal the new, higher volume of *harvesting*.

At this point, the system will be back in steady-state. However, because the *becoming mature* flow volume was less than the *harvesting* volume for six years, the stock of *Mature Trees* will have declined for six years.

And because *becoming mature* was less than *harvesting* by a constant amount, the decline will be *linear*. The *Mature Trees* stock will now rest at a *permanently lower* level than existed prior to the step-increase in *harvesting*.

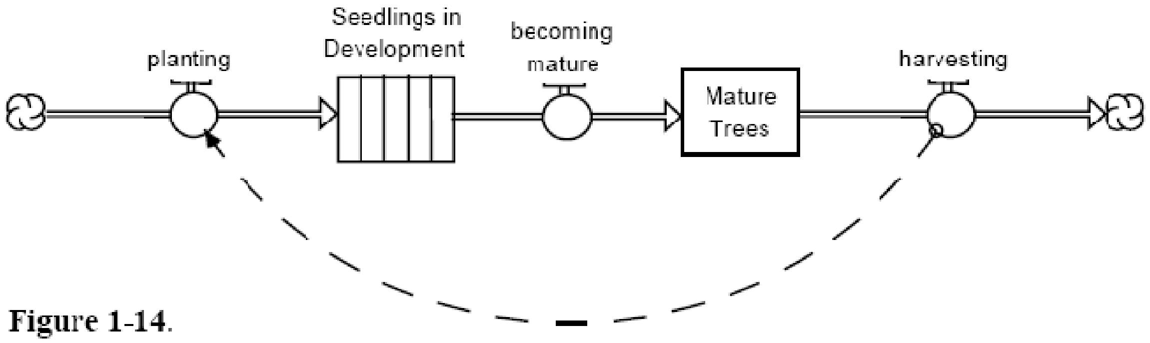


Figure 1-14.
A STELLA Map of the Tree-harvesting Story.

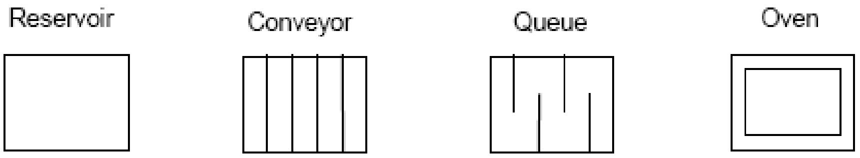


Figure 2-1.
The Four Types of Stock.

The Reservoir

The *reservoir* operates most like a real bathtub. Individual entities flow into a reservoir, and then become indistinguishable—just as individual water molecules flowing into a bathtub become indistinguishable (i.e., you can’t tell which molecule arrived first, which tenth, and which arrived last). Instead, the molecules blend together; all arrival time discipline and size-uniqueness are lost. You just have a certain number of liters of water in the tub. The same is true when you use a reservoir to represent, say, Population or Cash. You can’t distinguish Jamal from Janice in a reservoir labeled Population. You just have a total number of people. And the $100 bills are indistinguishable from the $1,000 bills in a reservoir named Cash. You just have a total amount of money. You can’t tell which bill came in when, nor can you distinguish bills of different denominations.

That’s what reservoirs do. They blur distinctions between the individual entities that flow into and out of them. Instead, they collect whatever *total* volume of stuff flows in, and give up whatever *total* volume flows out. At any point in time, they house the net of what has flowed in, minus what has flowed out.

The Conveyor

Think of conveyors as like those “moving sidewalks” at O’Hare or Heathrow airports. Or, conjure up an escalator at your favorite mall or department store. You step on either, you stand and ride for some distance, you get off—unless you’re one of those Type A’s who has to walk at full stride (while being transported) so as to at least *double* your ground speed. That’s how conveyors work. Whatever quantity arrives at the “first slat” gets on. It occupies the “first slat” on the conveyor. Nothing else can occupy that slat. The quantity “rides” until the conveyor deposits it “at the other end.” The “trip” will take a certain amount of time to complete (known as the “transit time”).

Conveyors are great for representing “pipeline delays” and all varieties of “aging chains.”

Unlike reservoirs, conveyors *do* maintain arrival integrity and, sometimes, also batch size. If one $100 bill arrives at the “first slat” at time 3, and one $500 bill arrives at time 5, you’d be able to distinguish the bills while they’re on the conveyor, and the $500 bill will “get off” two time units after the $100 bill—assuming the transit time of the conveyor remains constant (an assumption that can be relaxed—see the *Online Help Files* for details). Batch size is *not* retained in situations where, say, two $100 bills arrive at time 3 (you'd then simply have a total quantity of $200 "riding along").

The "danger" in relying too heavily on conveyors, a danger that heightens when employing queues and ovens, is loss of the 10,000 Meter viewpoint—a key viewpoint needed to do effective Systems Thinking. When you begin distinguishing between individual trucks, and worrying about whether that particular one (the red one over there) was delivered at 9:15 or 9:17, you have descended into the weeds and will no longer be able to see "the big picture." You're looking for specific answers, not general insights. You've traded your compass for a detailed street map. And you're also pushing the boundaries of what the *STELLA* software is best suited for doing. As a general rule, try to use reservoirs. If they really won't do the job, go with a conveyor. If you find yourself "going with a lot of conveyors," call us, we'll schedule you a "10,000 Meter" experience.

The Queue & The Oven

Frankly, we included these "mutants" in the software because the very technical end of the population using the software asked for them. These elements are pretty important for doing what's called "discrete event" simulations. Don't worry if this term is foreign to you. Suffice it to say that the *STELLA* software emanates out of a fundamentally "continuous" viewpoint on reality—again, we are talking the "10,000 meter" view. Queues and ovens serve the "discrete" worldview.
Including them in the software represents our attempt to do what physicists have been trying to do for 150 years—resolve the wave/particle duality issue! We figured, "No problem guys, here's the answer you've been looking for!"

Queues

This said, for certain applications, queues and ovens can be useful. So I'll briefly describe them here. A queue is a "line" like you often see waiting to check in at an airline ticket counter, or in front of our offices every morning waiting to purchase the *STELLA* software. Queues develop when things arrive at a rate that exceeds the capacity to "process" them. Think of cars stacking up at the tollbooths on the George Washington Bridge, waiting to enter New York City. Or, even closer to my own heart, cars amassing at one of the multiple entrances to what New Englanders affectionately refer to as "a rotary" (and I refer to as "the circle of death"). Ah, civility at its best!
Queues retain both arrival integrity and batch size. In the *STELLA*

software, queues enforce niceness. No “cutting in line” or “saving a place for a friend” is allowed. There’s also no “leaving” once you’re in line. When a volume of stuff “arrives,” if it can’t “get in/get on,” it sits in a queue (in a unique spot) until it can. Stuff that arrives later “gets in line” behind the stuff that’s already there. And it stays there! Again, you can visit the *Online Help Files* for more information on Queues.

Ovens

If conveyors are escalators, ovens are elevators. People arrive at an elevator, and if the doors happen to be open, they enter and then ride. In the more likely event that the doors are closed…people queue up, the car arrives, the doors open, people exit, the mob enters, the doors close (no one else can get on), and you ride. It’s the same in the *STELLA* software. Stuff arrives at an oven. If the oven is currently “baking,” the stuff waits (in a queue, or a reservoir). When the “baking cycle” is complete, it exits, and the stuff that’s waiting, enters (up to the capacity of the oven, or until the “doors open” time expires). That stuff then “bakes” for the length of the oven’s “bake time.” It’s then disgorged. The *Online Help Files* are once again your authoritative source for detail on oven operation.

Figure 2-2.
Two Flow Types and One "Wrinkle."

The Uniflow

The standard flow type is called a “uniflow,” which is short for “unidirectional.” The direction of flow is indicated by the arrowhead. If a uniflow points *into* a stock, it can only *fill* the stock—and vice versa. If a uniflow is an *inflow*, and for whatever reason, its calculated value during a simulation was a *negative* number (indicating that the flow should be *draining* the stock), the calculated value would be over-ridden by a value of *zero*! That is, inflows cannot operate as outflows! Another way to say this is, what you see is what you get! If the diagram shows it as an inflow…that’s how it works!.

The other kind of flow is the biflow (for “bi-directional”). It allows flow volume to go in *both* directions, either into or out of a stock. As you’ll discover when you learn how to “write sentences,” the general rule is that if the processes governing the inflow and outflow to a stock are identical in nature, use a biflow. Otherwise, use a uniflow. A good example of a legitimate biflow is “velocity.” If you had a stock called Distance, which represented the total number of kilometers you had traveled away from a starting point in, say, a Northerly direction, the associated *inflow* volume would be northbound velocity. Because you also can turn around (i.e., head Southward), and the process of generating South-bound velocity is identical (except for the direction you are headed in), velocity is correctly depicted as a biflow.